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Supergravity supertubes

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Abstract: We find the supergravity solution sourced by a supertube: a (1/4)-supersymmetric D0-charged IIA superstring that has been blown up to a cylindrical D2-brane by angular momentum. The supergravity solution captures all essential features of the supertube, including the D2-dipole moment and an upper bound on the angular momentum: violation of this bound implies the existence of closed timelike curves, with a consequent ghost-induced instability of supertube probes.

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Contents

1. Introduction

It has been recently shown by two of us that a cylindrical D2-brane can be supported against collapse by the angular momentum generated by electric and magnetic Born-Infeld fields [1]. These fields can be interpreted as some number q_s of dissolved IIA superstrings and some number per unit length q_0 of dissolved D0-branes. The angular momentum J is proportional to $q_0 q_s$ and the energy is minimized for a cylinder radius proportional to $\sqrt{q_0 q_s}$. Somewhat surprisingly, this configuration preserves 1/4 of the supersymmetry of the IIA Minkowski vacuum, hence the name *supertube*. As discussed in [1], and extended here to the case of $N > 1$ D2-branes,[1] one can also consider a combined system consisting of a supertube and D0-charged strings with given total string and D0 charges Q_s and Q_0, respectively. In this case supersymmetry does not fix the angular momentum but instead implies the upper bound $|J| \leq R|Q_s Q_0|^{1/2}$, with the radius of an N-times-wound supertube given by $R^2 = |J|/N$.

The gravitational back-reaction was not considered in [1], but when this is taken into account a supertube becomes a source for the IIA supergravity fields that govern the low-energy limit of the closed string sector of IIA superstring theory. Although

[1] Or to a single N-wound D2-brane; for the purposes of this paper both situations are equivalent.

this source is a distributional one (in the limit of weak string coupling), one might still expect it to generate a solution of IIA supergravity that is non-singular everywhere away from the source. Such a solution should carry the same string and D0-brane charges as the D2-brane supertube, as well as the same angular momentum, and preserve the same 1/4 of the supersymmetry of the IIA vacuum. The aim of this paper is to exhibit this solution, which we call the *supergravity supertube*, and to study its properties.

As we shall see, the supergravity solution accurately reproduces the features of the D2-brane supertube described above. In particular, the bound on the angular momentum arises from the requirement of causality: if J exceeds the bound then the Killing vector field associated with the angular momentum becomes timelike in a region near the supertube; since its orbits are closed this implies the existence of closed timelike curves (CTCs). This is a *global* violation of causality that cannot cause unphysical behaviour of any *local* probe (as we verify for a D0-brane probe) but it can and does cause unphysical behaviour for D2-brane probes, due to the appearance of a ghost-excitation on the D2-brane worldvolume.

Since the D2-brane supertube is a supersymmetric solution one might expect the force between parallel or concentric[2] supertubes to vanish, allowing them to be superposed. This is certainly the case for the D0-charged superstrings to which the supertube reduces in the limit of zero angular momentum, and it has been argued in the context of matrix theory [2] that it is also true of supertubes. As any force between parallel supertubes is transmitted by the IIA supergravity fields, it would seem necessary to consider the supergravity supertube solution to verify this, and this is one motivation for the present work. In fact, we shall exhibit 'multi-tube' solutions representing a number of parallel supertubes with arbitrary locations and radii, which implies the existence of a 'no-force' condition between parallel supertubes. By considering D0 and IIA string probes in this background we also establish a 'no-force condition' between supertubes and strings and D0-branes.

As noted in [1], the D2-brane supertube is T-dual to a helical rotating IIB D-string (the S-dual of which is T-dual to a helical rotating IIA string). These (1/4)-supersymmetric rotating helical strings have since been studied in two recent papers [3, 4]. A supergravity solution representing the asymptotic fields of a IIB rotating helical D-string was also presented in [4]. In fact, this solution is T-dual to a six-dimensional version of the supertube; we shall comment further on lower-dimensional supertubes in a concluding section. We should also mention that a IIA supergravity solution for a D2-brane tube with D0-branes and IIA string charges was constructed previously by one of the authors [5]; however, as already noted in [5] this solution does not describe a supertube because it has no angular momentum.[3]

[2]There is no topological distinction between these cases in a space of dimension ≥ 4.

[3]The results we present here suggest that the solution of [5] should be interpreted as a simple superposition of an unstable D2-brane tube with D0-charged strings.

2. Worldvolume supertubes

In this section we will review the results of [1] that are relevant to this paper, with a slight extension to allow for multiply-wound D2-branes. The conventions used here differ slightly from those of [1].

The starting point is a D2-brane in the ten-dimensional Minkowski vacuum of IIA superstring theory. We write the Minkowski metric as

$$ds^2_{10} = -dt^2 + dx^2 + dr^2 + r^2\, d\varphi^2 + ds^2(\mathbb{E}^6)\,, \tag{2.1}$$

where $\varphi \sim \varphi + 2\pi$. The induced metric g on a cylindrical D2-brane of constant radius R, at a fixed position in $\mathbb{E}^6$, aligned with the x-direction and with cross-section parametrized by φ, is

$$ds^2(g) = -dt^2 + dx^2 + R^2\, d\varphi^2\,, \tag{2.2}$$

where we have identified the worldvolume time with t. We will allow for a time-independent electric field E in the x-direction, and a time-independent magnetic field B, so the Born-Infeld 2-form is

$$F = E\, dt \wedge dx + B\, dx \wedge d\varphi\,. \tag{2.3}$$

The number of supersymmetries preserved by any brane configuration in a given spacetime is the number of independent Killing spinors ϵ of the background for which

$$\Gamma\epsilon = \epsilon\,, \tag{2.4}$$

where Γ is the matrix appearing in the 'κ-symmetry' transformation of the worldvolume spinors, its particular form depending on the background and on the type of brane. The spacetime Minkowski metric (2.1) may be written as

$$ds^2_{10} = -e^t e^t + e^x e^x + e^r e^r + e^\varphi e^\varphi + e^a e^a \tag{2.5}$$

for orthonormal 1-forms

$$e^t = dt\,, \qquad e^x = dx\,, \qquad e^r = dr\,, \qquad e^\varphi = r\, d\varphi\,, \qquad e^a = d\rho^a\,, \tag{2.6}$$

where $\{\rho^a\}$ are Cartesian coordinates on $\mathbb{E}^6$. Let $\Gamma_t, \Gamma_x, \Gamma_r, \Gamma_\varphi$ and $\{\Gamma_a\}$ be the ten constant tangent-space Dirac matrices associated to the above basis of 1-forms, and let $\Gamma_\natural$ be the constant matrix of unit square which anticommutes with all them. The Killing spinors in this basis take the form $\epsilon = M_+\, \epsilon_0$, where ϵ_0 is a constant 32-component spinor and

$$M_\pm \equiv \exp\left(\pm\frac{1}{2}\varphi\, \Gamma_{r\varphi}\right). \tag{2.7}$$

For the D2-brane configuration of interest here we have

$$\Gamma = \frac{1}{\sqrt{-\det(g+F)}}\left(\Gamma_{tx\varphi} + E\,\Gamma_{\varphi}\Gamma_{\natural} + B\,\Gamma_t\Gamma_{\natural}\right), \tag{2.8}$$

where

$$\sqrt{-\det(g+F)} = \sqrt{1-E^2+B^2}\,. \tag{2.9}$$

The condition for supersymmetry (2.4) thus becomes

$$M_+\left[B\,\Gamma_t\Gamma_{\natural} - \sqrt{-\det(g+F)}\right]\epsilon_0 + M_-\,\Gamma_{\varphi}\Gamma_{\natural}\left[\Gamma_{tx}\Gamma_{\natural} + E\right]\epsilon_0 = 0\,. \tag{2.10}$$

Since this equation must be satisfied for all values of φ, both terms must vanish independently. The vanishing of the second term requires that $E = \pm 1$ and $\Gamma_{tx}\Gamma_{\natural}\epsilon_0 = \mp\epsilon_0$. Without loss of generality we choose $E = 1$ and

$$\Gamma_{tx}\Gamma_{\natural}\epsilon_0 = -\epsilon_0\,. \tag{2.11}$$

Now the first term in (2.10) vanishes identically if $B = 0$, in which case we have 1/2 supersymmetry. We shall therefore assume that $B \neq 0$. In this case vanishing of the first term requires $\Gamma_t\Gamma_{\natural}\epsilon_0 = \pm\epsilon_0$ and $\mathrm{sign}(B) = \pm 1$. Again without loss of generality we shall assume that $B > 0$ and

$$\Gamma_t\Gamma_{\natural}\epsilon_0 = \epsilon_0\,. \tag{2.12}$$

The two conditions (2.11) and (2.12) on ϵ_0 are compatible and imply preservation of 1/4 supersymmetry. They are respectively associated with string charge along the x-direction and with D0-brane charge.

Under the conditions above, the D2-brane Lagrangian (for unit surface tension) is

$$\mathcal{L} = -\sqrt{R^2\left(1-E^2\right)+B^2}\,. \tag{2.13}$$

The momentum conjugate to E takes the form

$$\Pi \equiv \frac{\partial\mathcal{L}}{\partial E} = \frac{R^2\,E}{\sqrt{R^2\left(1-E^2\right)+B^2}}\,, \tag{2.14}$$

and the corresponding Hamiltonian density is

$$\mathcal{H} \equiv \Pi E - \mathcal{L} = R^{-1}\sqrt{\left(\Pi^2+R^2\right)\left(B^2+R^2\right)}. \tag{2.15}$$

The integrals

$$q_s \equiv \frac{1}{2\pi}\oint d\varphi\,\Pi \qquad \text{and} \qquad q_0 \equiv \frac{1}{2\pi}\oint d\varphi\,B \tag{2.16}$$

are (for an appropriate choice of units) the IIA string conserved charge and the D0-brane conserved charge per unit length carried by the tube. For a supersymmetric configuration $E = 1$ and B is constant, so from (2.14) and (2.16) we deduce that

$$R = \sqrt{|q_s q_0|}\,. \tag{2.17}$$

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The tension or energy per unit length of the tube is in turn

$$\tau = \frac{1}{2\pi} \oint d\varphi\, \mathcal{H}\,. \tag{2.18}$$

This is of course minimized at the supersymmetric radius (2.17), for which we find

$$\tau = |q_s| + |q_0|\,. \tag{2.19}$$

This result shows that the positive energies associated to D2-brane tension and rotation are exactly cancelled by the negative binding energy of the strings and D0-branes with the D2-brane, and hence that the supertube is a genuine bound state. As we shall see later, this has some counter-intuitive consequences.

The crossed electric and magnetic fields generate a Poynting 2-vector-density with

$$\mathcal{J}_\varphi = \Pi B \tag{2.20}$$

as its only non-zero component. The integral of $\mathcal{J}_\varphi$ over φ yields an angular momentum per unit length

$$J = q_s q_0 \tag{2.21}$$

along the axis of the cylinder. It is this angular momentum that supports the tube against collapse at the constant radius (2.17). In ten dimensions, the angular momentum 2-form L may have rank at most 8. This rank is 2 for the supertube, J being the only non-zero skew-eigenvalue of L. Note also that the angular momentum selects a 2-plane in the 8-dimensional space transverse to the strings, where the cross-section of the cylinder lies.

At this point we wish to consider a slight generalization of the results of [1] to allow for the possibilty of N coincident D2-brane tubes, or a single D2-brane supertube wound N times around the φ-circle, or combinations of coincident and multiply-wound D2-brane tubes. In any of these cases the local field theory on the D2-branes will be a U(N) gauge theory.[4] The results obtained in this paper will depend only on the total number N of D2-branes, so we need not distinguish between these possibilities; for convenience we shall consider the configuration of N D2-branes as a single N-times-wound D2-brane. In this case Π and B must be promoted to matrices in the Lie algebra of U(N), and their relationship to the string and D0-brane charges becomes

$$q_s \equiv \frac{1}{2\pi} \oint d\varphi\, \mathrm{Tr}\,\Pi\,, \qquad q_0 \equiv \frac{1}{2\pi} \oint d\varphi\, \mathrm{Tr}\,B\,. \tag{2.22}$$

[4]Provided that the radius of the tube is much larger than the string scale. Around the string scale a complex field coupling to the U(1) factor arising from strings connecting opposite points on the φ circle may become tachyonic, in which case the tube would decay to the vacuum with the emission of strings, D0-branes and massless closed string modes carrying the angular momentum; this is possible because the circle parameterized by φ is topologically trivial in spacetime.

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Since only their components along the identity contribute, we shall assume that only these components are non-zero and we shall still denote them by Π and B. We then have

$$q_s = N\Pi\,, \qquad q_0 = NB\,. \tag{2.23}$$

In the case that a single D2-brane winds N times around the φ-circle these relations are simply understood as due to the effective length of the D2-brane now being $2\pi RN$.

The hamiltonian density is now

$$\mathcal{H} = N\,R^{-1}\sqrt{(\Pi^2 + R^2)\,(B^2 + R^2)}, \tag{2.24}$$

which is still minimized by $R = \sqrt{|\Pi B|}$. However, in terms of the charges we now find

$$R = \frac{\sqrt{|q_s q_0|}}{N}\,, \tag{2.25}$$

and similarly for the angular momentum:

$$J = \frac{|q_s q_0|}{N}\,. \tag{2.26}$$

Note however that the tension is still given by (2.19).

As discussed in [1] for the $N = 1$ case, the quantity $|Q_s Q_0|/N$ is actually an *upper bound* on the angular momentum of a supersymmetric state with given string and D0-brane charges Q_s and Q_0. For

$$|J| \leq \frac{|Q_s Q_0|}{N} \tag{2.27}$$

there exist 'mixed' configurations consisting of an N-times-wound D2-brane supertube with charges q_s and q_0 such that $J = q_s q_0/N$, together with parallel IIA strings and D0-branes (and possibly D0-charged IIA strings) with charges q_s' and q_0' such that $Q_s = q_s + q_s'$ and $Q_s = q_0 + q_0'$. Since the D0-branes and strings carry no angular momentum, the combined system possesses total angular momentum $q_s q_0/N$, which is less than the maximal value allowed by the bound. By transferring charge from the strings and D0-branes to the supertube one can increase the angular momentum at fixed Q_0 and fixed Q_s, while maintaining the 1/4 supersymmetry. However, this process can only be continued until $q_s = Q_s$ and $q_0 = Q_0$. If one were to continue beyond this point, obtaining a configuration with $|q_s| > |Q_s|$, then q_s and q_s' would have different signs, and their associated supersymmetry projectors would have no common eigenspinors.

Thus we conclude that the angular momentum of a supersymmetric state of IIA string theory with charges Q_s and Q_0 is bounded from above as in (2.27). Since the supersymmetric radius of the supertube is always

$$R^2 = \frac{|J|}{N}\,, \tag{2.28}$$

the bound on the angular momentum may be rewritten as

$$J^2 \leq R^2 \left| Q_s Q_0 \right| . \tag{2.29}$$

3. Supergravity supertubes

Our starting point will be a solution of D=11 supergravity found in [6] that describes the intersection of two rotating M5-branes and an M2-brane, with an M-wave along the string intersection. Although the generic solution of this type preserves only 1/8 of the 32 supersymmetries of the M-theory vacuum, the special case in which we set to zero the M5-brane charges preserves 1/4 supersymmetry. The metric and 4-form field strength of this solution are (in our conventions)

$$\begin{aligned} ds^2_{11} &= U^{-2/3} \Big[- dt^2 + dz^2 + K\,(dt+dz)^2 + 2\,(dt+dz)\,A + dx^2 \Big] + U^{1/3} d\vec{y} \cdot d\vec{y} \,, \\ F_4 &= dt \wedge dU^{-1} \wedge dx \wedge dz - (dt+dz) \wedge dx \wedge d\,(U^{-1}A) \,. \end{aligned} \tag{3.1}$$

Here $\vec{y} = \{y^i\}$ are cartesian coordinates on $\mathbb{E}^8$. The membrane extends along the x- and z-directions, and the wave propagates along the z-direction. The functions U and K and the 1-form A depend only on $\vec{y}$. U and K are harmonic on $\mathbb{E}^8$ and are associated with the membrane and the wave, respectively. A determines the angular momentum of the solution, and its field strength dA satisfies the source-free Maxwell-like equation

$$d *_8 dA = 0 \,, \tag{3.2}$$

where $*_8$ is the Hodge dual operator on $\mathbb{E}^8$. The reduction to ten dimensions of this solution along the z-direction yields a (1/4)-supersymmetric IIA supergravity solution:

$$ds^2_{10} = -U^{-1}V^{-1/2}\,(dt - A)^2 + U^{-1}V^{1/2}\,dx^2 + V^{1/2}\,d\vec{y} \cdot d\vec{y} \,, \tag{3.3}$$

$$B_2 = -U^{-1}\,(dt - A) \wedge dx + dt \wedge dx \,, \tag{3.4}$$

$$C_1 = -V^{-1}\,(dt - A) + dt \,, \tag{3.5}$$

$$C_3 = -U^{-1} dt \wedge dx \wedge A \,, \tag{3.6}$$

$$e^{\phi} = U^{-1/2} V^{3/4} \,, \tag{3.7}$$

where $V = 1 + K$, and B_2 and C_p are the Neveu-Schwarz and Ramond-Ramond potentials, respectively, with gauge-invariant field strengths

$$H_3 = dB_2 \,, \qquad F_2 = dC_1 \,, \qquad G_4 = dC_3 - dB_2 \wedge C_1 \,. \tag{3.8}$$

We have chosen a gauge for the potentials B_2, C_1 and C_3 such that they all vanish at infinity if U and V are chosen such that the metric at infinity is the D=10 Minkowski metric in canonical Cartesian coordinates. We note here for future use that

$$G_4 = U^{-1}V^{-1}\,(dt - A) \wedge dx \wedge dA \,. \tag{3.9}$$

The Killing spinors of the solution (3.5) follow from those in eleven dimensions given in [6], but we have also verified them directly in D=10 with the conventions of [7]. They take a simple form when the metric is written as

$$ds^2_{10} = -e^t e^t + e^x e^x + e^i e^i \,, \tag{3.10}$$

with the orthonormal 1-forms

$$e^t = U^{-1/2} V^{-1/4} \,(dt - A)\,, \qquad e^x = U^{-1/2} V^{1/4}\, dx\,, \qquad e^i = V^{1/4}\, dy^i\,. \tag{3.11}$$

In this basis, the Killing spinors become

$$\epsilon = U^{-1/4} V^{-1/8}\, \epsilon_0\,, \tag{3.12}$$

where ϵ_0 is a constant 32-component spinor subject to the constraints (2.11) and (2.12). As mentioned before, these two constraints preserve 1/4 supersymmetry and are those associated with a IIA string charge aligned with the x-axis and D0-brane charge, as required for a supertube, although we still must specify the functions U and V and the 1-form A before we can make this identification.

To motivate the choice for these functions, consider first the solution describing D0-charged fundamental strings located at a point $\vec{y} = \vec{y}_\alpha$ in $\mathbb{E}^8$ and aligned with the x-axis. This solution is given by (3.5) with

$$U = 1 + \frac{|Q_s|}{6\Omega\, |\vec{y}-\vec{y}_\alpha|^6}\,, \qquad V = 1 + \frac{|Q_0|}{6\Omega\, |\vec{y}-\vec{y}_\alpha|^6}\,, \qquad A = 0\,, \tag{3.13}$$

where Ω is the volume of the unit 7-sphere. The constant Q_s is the string charge, while Q_0 is the D0-brane charge per unit length; this can be seen from their asymptotic contributions to their respective field strengths H_3 and F_2. (The signs of these charges are flipped by taking $t \to -t$ and/or $x \to -x$.)

Now consider distributing the charged strings homogeneously on a circle of radius R in a 2-plane in $\mathbb{E}^8$. Let r, φ be polar coordinates on this plane, and ρ the radial coordinate on the orthogonal 6-plane (see figure 1). The metric then takes the form

$$ds^2(\mathbb{E}^8) = dr^2 + r^2 d\varphi^2 + d\rho^2 + \rho^2 d\Omega_5^2\,, \tag{3.14}$$

where $d\Omega_5^2$ is the SO(6)-invariant metric on the unit 5-sphere. The solution for this configuration is still as in (3.5) with $A=0$, but with the harmonic functions obtained by linear superposition on the circle of those in (3.13), that is,

$$\begin{aligned} U(r,\rho) &= 1 + \frac{|Q_s|}{6\Omega}\,\frac{1}{2\pi}\int_0^{2\pi} d\alpha\, \frac{1}{|\vec{y}-\vec{y}_\alpha|^6} \\ &= 1 + \frac{|Q_s|}{6\Omega}\,\frac{1}{2\pi}\int_0^{2\pi} d\alpha\, \frac{1}{(r^2+\rho^2+R^2-2Rr\,\cos\alpha)^3}\,, \end{aligned} \tag{3.15}$$

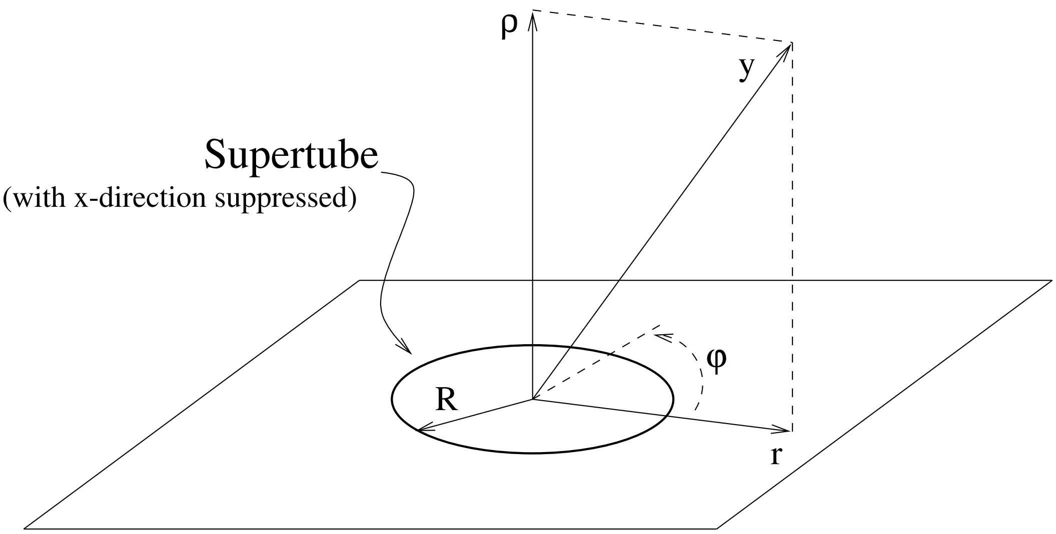

Figure 1: Coordinates on $\mathbb{E}^8$.

and similarly for V with Q_s replaced by Q_0. Thus

$$
\begin{aligned}
U &= 1 + \frac{|Q_s|}{6\Omega}\,\frac{\left(r^2+\rho^2+R^2\right)^2 + 2R^2r^2}{\Sigma^5}\,,\\
V &= 1 + \frac{|Q_0|}{6\Omega}\,\frac{\left(r^2+\rho^2+R^2\right)^2 + 2R^2r^2}{\Sigma^5}\,,
\end{aligned}
\tag{3.16}
$$

where

$$
\Sigma(r,\rho) = \sqrt{\left(r^2+\rho^2+R^2\right)^2 - 4R^2r^2}\,. \tag{3.17}
$$

Note that (by construction) these functions satisfy Laplace's equation on $\mathbb{E}^8$ with Dirac delta-like sources with support on the $r = R, \rho = 0$ circle. In addition, they have the same asymptotic behaviour as those in (3.13) in the limit $|\vec{y}| \to \infty$. Hence the constants Q_s and Q_0 in (3.16) are again the string and D0 charges.

The solution (3.5) with $A = 0$ and U and V as in (3.16) displays a tubular structure but possesses no angular momentum, and the field strength G_4 sourced by D2-branes vanishes. To describe the supertube we must incorporate the angular momentum, with the source of rotation being located at the tube. We know that the asymptotic form must be

$$
A \sim \frac{1}{2}\frac{L_{ij}\,y^j}{\Omega\,|\vec{y}|^8}\,dy^i\,, \tag{3.18}
$$

where the constants $L_{ij} = -L_{ji}$ are the components of the angular momentum 2-form L. As explained above, L must have rank 2 for the supertube, in which case we may write the asymptotic form as

$$
A \sim \frac{J}{2\Omega|\vec{y}|^8}\,r^2 d\varphi\,, \tag{3.19}
$$

where J is a constant angular momentum, the one non-zero skew-eigenvalue of L. The calculation for the exact form of A sourced by the supertube is essentially the same as that giving the vector potential created by a circular electric current of intensity proportional to J/R^2, that is,

$$A = \frac{1}{6\Omega}\,\frac{J}{2\pi R}\,r d\varphi \int_0^{2\pi} d\alpha\,\frac{\cos\alpha}{\left(r^2+\rho^2+R^2-2Rr\,\cos\alpha\right)^3}. \tag{3.20}$$

The result is

$$A = J\,\frac{\left(r^2+\rho^2+R^2\right)\,r^2}{2\Omega\,\Sigma^5}\,d\varphi\,, \tag{3.21}$$

which has the correct asymptotic behaviour (3.19). As we shall see in the next section, this choice of A automatically generates the correct D2-fields.

The supertube solution is thus given by (3.5) with U and V as in (3.16) and A as in (3.21); we shall analyze its properties in detail below. However, we wish to note here that evaluation of the ADM integral for brane tension [8] (in the same conventions as for the angular momentum) yields the tube tension

$$\tau = |Q_s| + |Q_0|\,, \tag{3.22}$$

exactly as for the D2-brane supertube (no factors of the string coupling constant appear in this formula because the asymptotic value of the dilaton vanishes for our chosen solution).

Having completed our construction of the supertube solution, we close this section by presenting its generalization to a 'multi-tube' solution representing N parallel tubes with arbitrary locations, radii and charges. These are easily constructed because of the linearity of the harmonicity conditions on U and V and of the Maxwell equation for A. The general expression is

$$\begin{aligned}
U &= 1+\sum_{n=1}^{N}\frac{Q_s^{(n)}}{6\Omega}\,\frac{\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2+2R_n^2|\vec{r}-\vec{r}_n|^2}{\left[\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2-4R_n^2|\vec{r}-\vec{r}_n|^2\right]^{5/2}}\,,\\
V &= 1+\sum_{n=1}^{N}\frac{Q_0^{(n)}}{6\Omega}\,\frac{\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2+2R_n^2|\vec{r}-\vec{r}_n|^2}{\left[\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2-4R_n^2|\vec{r}-\vec{r}_n|^2\right]^{5/2}}\,,\\
A &= \sum_{n=1}^{N}\frac{J^{(n)}}{2\Omega}\,\frac{\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2}{\left[\left(|\vec{y}-\vec{y}_n|^2+R_n^2\right)^2-4R_n^2|\vec{r}-\vec{r}_n|^2\right]^{5/2}}\left[(u-u_n)dv-(v-v_n)du\right]\,.
\end{aligned} \tag{3.23}$$

Here u and v are cartesian coordinates on the 2-plane selected by the angular momentum. All the tubes are aligned along the x-direction and all their cross-sections are parallel to each other. The n-th tube has radius R_n and is centred at

$$\vec{y} = (\vec{r}_n,\vec{\rho}_n) = (u_n, v_n, \vec{\rho}_n) \tag{3.24}$$

in $\mathbb{E}^8$. It carries string and D0 charges $Q_s^{(n)}$ and $Q_0^{(n)}$, respectively, and angular momentum $J^{(n)}$. The total charges and angular momentum are the sums of those carried by each tube. Clearly, by setting some of the radii and angular momenta to zero, we obtain a solution representing a superposition of D0-charged strings and supertubes.

The existence of this multitube solution shows that there is no force between stationary parallel supertubes.[5] Note that, as far as the supergravity solution is concerned, two (or more) tubes can intersect each other, as described by the solution above when the radii and centres are chosen appropriately.

4. D2-dipole structure and closed timelike curves

We have now found a (1/4)-supersymmetric solution of IIA supergravity that carries all the charges required for its interpretation as the solution sourced by a supertube source. Although it also displays the appropriate tubular structure, we have not yet identified clearly the presence of a D2-brane. Even though there cannot be any D2-brane charge, we would still expect the fields of the D2-brane supertube to carry a non-zero D2-brane dipole moment determined by the size of the tube. There is certainly a local D2-brane charge distribution because the electric components of G_4 are non-zero. Specifically, for A given by (3.21), we have, asymptotically,

$$G_4 = d\left[\frac{Jr^2}{2\Omega(r^2+\rho^2)^4}\,dt\wedge dx\wedge d\varphi\right] + \cdots\,, \tag{4.1}$$

where the dots stand for subleading terms in an expansion in $1/|\vec{y}|$ for $|\vec{y}|\to\infty$. The integral of $*G_4$ over any 6-sphere at infinity vanishes, showing that there is no D2-brane net charge. However, the expression (4.1) has the correct form to be interpreted as the dipole field sourced by a cylindrical D2-brane aligned with the x-axis; the scale of the dipole moment is set by the angular momentum,

$$\mu_2 \sim J\,. \tag{4.2}$$

In turn, the dipole moment must be related to the size of the source as

$$|\mu_2| \sim NR^2\,, \tag{4.3}$$

where R is the radius of the D2-tube and N the number of D2-branes. This can be seen as follows. The dipole moment of N spherical D2-branes of radius R scales as $|\mu|\sim NR^3$ [9]. For a cylindrical D2-brane of length a this must be replaced by

[5] Nor between supertubes and strings or D0-branes. We shall confirm and elaborate on this in section 5.

$|\mu| \sim NaR^2$. In the present situation a is infinite, so μ_2 in equations (4.2) and (4.3) is actually a dipole moment per unit length. Thus, we conclude that

$$R^2 \sim \frac{|J|}{N}\,, \tag{4.4}$$

in agreement with the worldvolume analysis of section 2. This agreement could be made more precise by fixing the proportionality factors in the relations above. Instead, we now turn to analyzing the solution near the tube, which will yield the same result and provide additional information on the detailed structure of the supertube, in particular concerning the presence and the number of D2-branes.

To this effect, we perform a change of coordinates which is convenient to focus on the region close to the tube:

$$\begin{aligned} r &= \sqrt{(\hat{r}\cos\hat{\theta} + R)^2 - \hat{r}^2}\,, \\ \rho &= \hat{r}\sin\hat{\theta}\,. \end{aligned} \tag{4.5}$$

This has been designed so that, in the new coordinates, $\Sigma = 2R\hat{r}$. In the limit

$$\hat{r}/R \ll 1 \tag{4.6}$$

one approaches the tube at $r = R$, $\rho = 0$. Note that this can be achieved by either fixing R and making $\hat{r}$ small, or by fixing $\hat{r}$ and making R large. In the first case one focuses, for a given solution, on the region near the tube. In the second, the radius of the tube grows very large while we remain at a finite distance from it. In both cases the tube looks planar. In this limit the metric becomes

$$\begin{aligned} ds^2 = &-U^{-1}V^{-1/2}\,(dt - k\,d\hat{z})^2 + U^{-1}V^{1/2}\,dx^2 + \\ &+V^{1/2}\left(d\hat{z}^2 + d\hat{r}^2 + \hat{r}^2 d\hat{\theta}^2 + +\hat{r}^2\sin^2\hat{\theta} d\Omega_5^2\right)\,, \end{aligned} \tag{4.7}$$

where we have defined $\hat{z} = R\varphi$, so $\hat{z}$ is a coordinate identified with period $2\pi R$. The three functions in the metric above are

$$\begin{aligned} U &= 1 + \frac{|Q_s|/2\pi R}{5\Omega_6 \hat{r}^5} + \cdots\,, \\ V &= 1 + \frac{|Q_0|/2\pi R}{5\Omega_6 \hat{r}^5} + \cdots\,, \\ k &= \frac{J/2\pi R^2}{5\Omega_6 \hat{r}^5} + \cdots\,, \end{aligned} \tag{4.8}$$

where the dots stand for subleading $\hat{r}$-dependent terms in the limit (4.6). Here Ω_6 is the volume of the unit 6-sphere. The gauge potentials are as in (3.5) with U and V given above and $A = kd\hat{z}$.

The solution in the form (4.7) clearly exhibits the properties expected from the planar limit close to the supertube. The angular momentum becomes linear

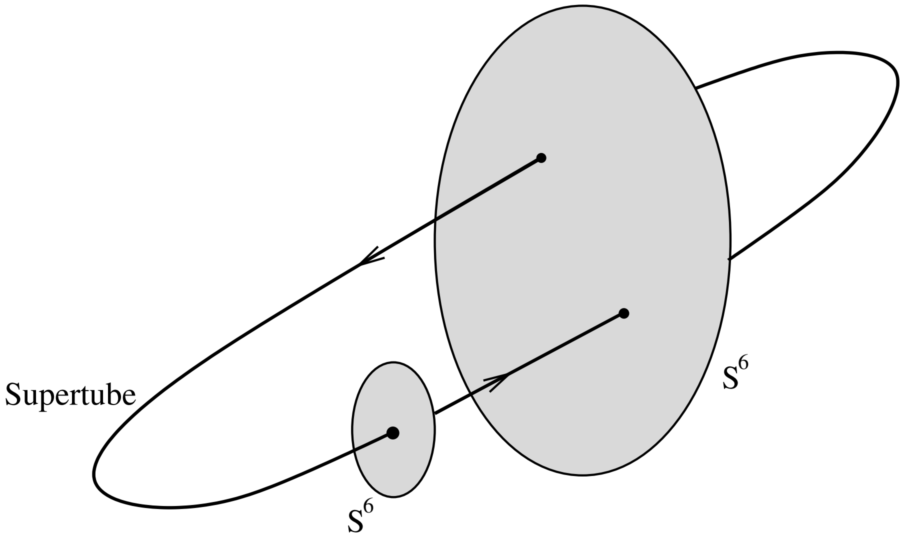

Figure 2: The little 7-disc intersects the supertube (suppressing the x-direction) at one point, hence the flux of $*G_4$ through its boundary (the little S^6) is non-zero. On the contrary, the large disc intersects the supertube at two points which correspond to local D2-brane charge densities of opposite sign, hence the flux through the large S^6 vanishes; the same happens for any 6-sphere at infinity.

momentum along the tangent direction to the circle, that is, along $\hat{z}$. The SO(6)-symmetry associated to rotations in $\mathbb{E}^6$ is enhanced to SO(7): the $\hat{\theta}$-coordinate in the near-tube metric combines with the coordinates on the 5-sphere to yield the metric on a round 6-sphere. The functions U and V are sourced by delta-functions at $\hat{r} = 0$, hence the gauge potentials B_2 and C_1 correspond to charge densities $Q_s/2\pi R$ and $Q_0/2\pi R$ along the $\hat{z}$-direction. The four-form field strength

$$G_4 = -U^{-1}V^{-1}\, dt \wedge dx \wedge d\hat{z} \wedge dk \tag{4.9}$$

corresponds to the charge density of $N = |J|/R^2$ D2-branes at $\hat{r} = 0$, in agreement with (2.28). To see this, consider a 7-disc that is small enough to intersect the D2-circle at only one point (see figure 2) and compute the flux of $*G_4$ through its 6-sphere boundary. The result is precisely

$$N \equiv \left| 2\pi \int_{S^6} *G_4 \right| = \frac{|J|}{R^2}\,. \tag{4.10}$$

As the radius of the 6-sphere increases, the 7-disc will eventually intersect the D2-circle at a second point and the above integral then vanishes, as happens for any 6-sphere at infinity.[6]

[6]It follows from the considerations above that the limit $R \to \infty$ with fixed charge densities results in a planar configuration of D2-branes with strings and D0-branes distributed on it, and with momentum in the direction transverse to the strings. This preserves the same amount of supersymmetries as the supertube. To our knowledge, this configuration has not been previously considered.

We have now identified all the elements present in the supertube solution. There are four independent parameters, say Q_s, Q_0, R and N (or J). So far, the solution displays all the features of the worldvolume supertube except for the bound (2.29). We shall now show that this arises by demanding that there be no causality violations in the supertube spacetime.

Consider the Killing vector field $\ell \equiv \partial/\partial\varphi$, which is associated to rotations on the 2-plane selected by the angular momentum. Its norm squared is

$$|\ell|^2 = g_{\varphi\varphi} = U^{-1}V^{-1/2}r^2 \left(UV - f^2/r^2\right), \tag{4.11}$$

where

$$f = J\,\frac{(r^2+\rho^2+R^2)\; r^2}{2\Omega\,\Sigma^5}\,. \tag{4.12}$$

The norm of ℓ is always positive for large enough $|\vec{y}|$, regardless of the value of J, and if the bound (2.29) is satisfied then ℓ remains spacelike everywhere. However, if J exceeds the bound then ℓ becomes timelike sufficiently close to the tube. To see this, we write out (4.11) as

$$\begin{aligned}|\ell|^2 = \frac{U^{-1}V^{-1/2}r^2}{36\Omega^2\Sigma^{10}}\Big\{ & 36\Omega^2\Sigma^{10}+6\Omega\Sigma^5(|Q_0|+|Q_s|)\left[(r^2+\rho^2+R^2)^2+2R^2r^2\right]+|Q_0Q_s|\times \\ & \times\Big[(r^2-R^2)^2(r^4+r^2R^2+R^4)+2\rho^2(r^2+R^2)\times \\ & \quad\times\left[2(r^2-R^2)^2+3r^2R^2\right]+\rho^4\left[6(r^2-R^2)^2+19R^2r^2\right]+ \\ & \quad+4\rho^6(r^2+R^2)+\rho^8\Big]+9(R^2|Q_sQ_0|-J^2)(r^2+R^2+\rho^2)^2r^2\Big\}\,.\end{aligned}$$

All the terms in this expression are non-negative except for the last one, which becomes negative precisely when the bound (2.29) is violated. In addition, in the near-tube limit

$$r \to R\,, \qquad \rho \to 0 \tag{4.13}$$

we find

$$|\ell|^2 = \frac{R^2}{\Omega^{1/2}\Sigma^{5/2}|Q_s||Q_0|^{1/2}}\left[\left(R^2|Q_sQ_0|-J^2\right)+\cdots\right], \tag{4.14}$$

where the dots represent terms which vanish in the limit (4.13). Thus the supergravity supertube has CTCs if and only if it is 'over-rotating'. These CTCs are 'naked' in the sense that there is no event horizon to prevent them from being deformed to pass through any point of the spacetime. Neither can they be removed by a change of frame, since the causal structure is unchanged by any non-singular conformal rescaling of the metric. It should be appreciated that the surface defined by $|\ell|^2 = 0$ (present when J exceeds the bound) is *not* singular; it is not even a coordinate singularity. In particular, the metric signature does *not* change on this surface despite the fact that for $|\ell|^2 < 0$ there are two commuting timelike Killing vector fields. In fact, the only physical singularity of the solution (3.5) is at $r = R, \rho = 0$, where both $|\ell|^2$ and the dilaton diverge. Thus, the over-rotating supertube spacetime is *locally* as physical as the under-rotating one, with a metric that is singular only at the location of the tube.

The CTCs of the over-rotating supertube metric have a very simple origin in eleven dimensions. The IIA supergravity supertube lifts to the solution (3.1) of eleven-dimensional supergravity from which we started, with U and V as in (3.16) and A as in (3.21). If the eleventh coordinate z is not periodically identified then this eleven-dimensional spacetime has no CTCs, but CTCs are created by the identifications needed for compactification on a circle if J exceeds the bound (2.29). To see this, consider an orbit of the vector field $\xi = \partial_\varphi + \beta\partial_z$, for real constant β. When z is periodically identified this curve will be closed for some dense set of values of β. It will also be timelike if $|\xi|^2 < 0$, which will happen if

$$\beta^2 + 2V^{-2}f\,\beta + UV^{-2}r^2 < 0\,. \tag{4.15}$$

This inequality can be satisfied for real β only if $r^2 < U^{-1}V^{-1}f^2$, but this is precisely the condition that ℓ be timelike. Thus, there will be closed timelike orbits of ξ in the D=11 spacetime precisely when the closed orbits of ℓ become timelike in the D=10 spacetime. However, from the eleven-dimensional perspective, these CTCs arise from periodic identification and can therefore be removed by passing to the universal covering space. An analogous phenomenon was described in [10], where it was shown that the CTCs of over-rotating supersymmetric five-dimensional black holes are removed by lifting the solution to ten dimensions and passing to the universal covering space.

5. Brane probes: supersymmetry

We shall now examine the behaviour of brane probes in the supertube spacetime. Specifically, we shall consider D0-branes, strings and D2-branes. We shall first show that the 1/4 supersymmetry of the background is preserved by stationary D0-brane, IIA-string and D2-supertube probes (when suitably aligned).

The D=10 spacetime metric (3.5) can be written as

$$ds^2_{10} = -e^te^t + e^xe^x + e^re^r + e^\varphi e^\varphi + e^ae^a\,, \tag{5.1}$$

for orthonormal 1-forms

$$\begin{aligned} e^t &= U^{-1/2}V^{-1/4}\,(dt - A)\,, \\ e^x &= U^{-1/2}V^{1/4}\,dx\,, \\ e^r &= V^{1/4}\,dr\,, \\ e^\varphi &= V^{1/4}\,r\,d\varphi\,, \\ e^a &= V^{1/4}\,d\rho^a\,. \end{aligned} \tag{5.2}$$

Here $\{\rho^a\}$ are cartesian coordinates on the $\mathbb{E}^6$ space orthogonal to the 2-plane selected by the angular momentum. The Killing spinors in this basis take the form

$$\epsilon = U^{-1/4} V^{-1/8} \, M_+ \, \epsilon_0 \,, \tag{5.3}$$

where ϵ_0 is a constant 32-component spinor subject to the constraints (2.11) and (2.12). Recall that we introduced the matrices $\Gamma_t, \Gamma_x, \Gamma_r, \Gamma_\varphi, \{\Gamma_a\}, \Gamma_\natural$ and $M_\pm$ in section 2. Recall too that the number of supersymmetries preserved by any brane configuration in a given spacetime is the number of independent Killing spinors of the background verifying (2.4).

For a stationary D0-brane in the gauge in which worldline time is identified with t we have

$$\Gamma_{D0} = \Gamma_t \Gamma_\natural \,. \tag{5.4}$$

For a IIA string in the same temporal gauge and, additionally, with the string coordinate identified with x, we have

$$\Gamma_{string} = -\Gamma_{tx} \Gamma_\natural \,. \tag{5.5}$$

These two Γ-matrices commute and the Killing spinors of the background are simultaneous eigenstates of Γ_{D0} and Γ_{string} with unit eigenvalue, so the inclusion of these D0-brane and IIA-string probes does not break any supersymmetries that are preserved by the background. In particular, this means that a D0-brane or a string can be placed arbitrarily close to the supertube without experiencing any force. This result is counter-intuitive because one would expect the D2-cylinder in this situation to behave approximately like a planar D2-brane, and hence to exert an attractive force on D0-branes and strings. The explanation is presumably that the supertube is in fact a true bound state of strings, D0-branes and cylindrical D2-branes, in which the D2-branes behave genuinely differently as compared to their free counterparts.[7]

Now we consider a probe consisting of a supertube itself, that is, a D2-brane of cylindrical topology with string and D0-brane charges. The analysis is very similar to that of section 2 except that the Born-Infeld field strength F must now be replaced by the background-covariant field strength $\mathcal{F} = F - B_2$. As before we choose F to have the form

$$F = E\, dt \wedge dx + B\, dx \wedge d\varphi \,. \tag{5.6}$$

This yields

$$\mathcal{F} = \mathcal{E}\, dt \wedge dx + \mathcal{B}\, dx \wedge d\varphi \,, \tag{5.7}$$

where

$$\mathcal{E} = E + U^{-1} - 1 \,, \qquad \mathcal{B} = B + U^{-1} f \,. \tag{5.8}$$

[7] The neutron considered as a bound state of quarks is analogous since the attractive force between neutrons is much weaker than the force between its constitutive quarks.

In the orthonormal basis (5.2) we have

$$\mathcal{F} = \bar{\mathcal{E}}\, e^t \wedge e^x + \bar{\mathcal{B}}\, e^x \wedge e^\varphi \,, \tag{5.9}$$

where

$$\bar{\mathcal{E}} = U\mathcal{E}\,, \qquad \bar{\mathcal{B}} = U^{1/2}V^{-1/2}r^{-1}\,(\mathcal{B} - f\mathcal{E})\,. \tag{5.10}$$

In terms of these variables, Γ_{D2} takes the same form as in flat space:

$$\Gamma_{D2} = \frac{1}{\sqrt{-\det(g+\mathcal{F})}} \left(\Gamma_{tx\varphi} + \bar{\mathcal{E}}\,\Gamma_\varphi \Gamma_\natural + \bar{\mathcal{B}}\,\Gamma_t \Gamma_\natural \right), \tag{5.11}$$

where

$$\sqrt{-\det(g+\mathcal{F})} = \sqrt{1 - \bar{\mathcal{E}}^2 + \bar{\mathcal{B}}^2}\,. \tag{5.12}$$

The condition for supersymmetry (2.4) thus becomes

$$M_+ \left[\bar{\mathcal{B}}\,\Gamma_t \Gamma_\natural - \sqrt{-\det(g+\mathcal{F})} \right] \epsilon_0 + M_-\,\Gamma_\varphi \Gamma_\natural \left[\Gamma_{tx}\Gamma_\natural + \bar{\mathcal{E}} \right] \epsilon_0 = 0\,. \tag{5.13}$$

Both terms must vanish independently. Given the constraints (2.11) and (2.12) satisfied by ϵ_0, the vanishing of the second term requires that $\bar{\mathcal{E}} = 1$. Equivalently,

$$\mathcal{E} = U^{-1}\,. \tag{5.14}$$

The first term in (5.13) then vanishes identically if $\bar{\mathcal{B}} \geq 0$, which using (5.14) is equivalent to $B \geq 0$. In the $B = 0$ case we have 1/2 supersymmetry, so we shall assume that

$$B > 0\,. \tag{5.15}$$

6. Brane probes: energetics

We now turn to the energetics of probes in the supertube spacetime. Our aim is to uncover any effects of the presence of CTCs on brane probes, so in this section we shall assume that the bound (2.29) is violated. We shall see that the *global* causality violation due to the CTCs causes a *local* instability on the worldvolume of extended probes such as a D2-brane tube, the reason for this being of course that the probe itself is non-local. On the contrary, we would not expect any unphysical effect on a local probe such as a D0-brane, and we shall begin by verifying this.

The action for a D0-brane of unit mass is

$$S_{D0} = -\int e^{-\phi}\sqrt{-\det g} - \int C_1\,. \tag{6.1}$$

In the gauge where the worldline time is identified with t we find the lagrangian density

$$\mathcal{L} = -V^{-1}\sqrt{1 - 2f\,\dot{\varphi} - UV^{1/2}\left(g_{\varphi\varphi}\,\dot{\varphi}^2 + v^2\right)} + V^{-1}(1 - f\dot{\varphi}) - 1\,, \tag{6.2}$$

where the overdot indicates differentiation with respect to t, f is given by (4.12), and

$$v^2 \equiv g_{ij}\dot{X}^i\dot{X}^j\,, \qquad X^i = \{x, r, \rho^a\}\,. \tag{6.3}$$

Note that $v^2 \geq 0$.

The lagrangian (6.2) appears to lead to unphysical behaviour if $g_{\varphi\varphi}$ becomes negative. However, an expansion in powers of the velocities yields

$$\mathcal{L} = -1 + \frac{1}{2}Ur^2\,\dot{\varphi}^2 + \frac{1}{2}UV^{-1/2}v^2 + \cdots\,. \tag{6.4}$$

The first term is minus the (unit) positive mass of the D0-brane. As expected from the supersymmetry of a stationary D0-brane, all the velocity-independent potential terms cancel. In addition, the kinetic energy is positive-definite regardless of the sign of $g_{\varphi\varphi}$ (and this remains true to all orders in the velocity). There is therefore nothing to prevent a D0-brane from entering the region where $|\ell|^2 < 0$, and its dynamics in this region is perfectly physical, at least locally. The same applies to x-aligned IIA superstrings; again this is not surprising because this is a probe that is local in φ.

We now turn to the supertube itself. This is a more significant test of the geometry because one could imagine building up the source of the supergravity solution by accretion of concentric D2-brane supertubes. We begin by considering a general tubular D2-brane aligned with the background. For our purposes we may assume that the worldvolume scalar fields r and ρ^a are uniform in x, but we shall also assume, initially, that they are time-independent (as will be the case for a supertube, since this is a D2-brane tube at a minimum of the potential energy).

The action

$$S_{D2} = -\int e^{-\phi}\sqrt{-\det(g+\mathcal{F})} - \int (C_3 + C_1 \wedge \mathcal{F}) \tag{6.5}$$

in the physical gauge (as used above) yields the Lagrangian density

$$\mathcal{L} = -U^{1/2}V^{-1}\sqrt{Vr^2(U^{-2}-\mathcal{E}^2) + U^{-1}(\mathcal{B}-f\mathcal{E})^2} + V^{-1}(\mathcal{B}-f\mathcal{E}) - B\,. \tag{6.6}$$

The variable conjugate to E is

$$\Pi \equiv \frac{\partial\mathcal{L}}{\partial E} = \frac{\partial\mathcal{L}}{\partial\mathcal{E}} = \frac{U^{1/2}r^2\mathcal{E} + U^{-1/2}V^{-1}(\mathcal{B}-f\mathcal{E})f}{\sqrt{Vr^2(U^{-2}-\mathcal{E}^2) + U^{-1}(\mathcal{B}-f\mathcal{E})^2}} - V^{-1}f\,. \tag{6.7}$$

The hamiltonian density is defined as

$$\mathcal{H} \equiv \Pi E - \mathcal{L} = \Pi\mathcal{E} - \mathcal{L} + \Pi(1-U^{-1})\,. \tag{6.8}$$

For supersymmetric configurations we have $\mathcal{L} = -B$ and $\mathcal{E} = U^{-1}$, so[8]

$$\mathcal{H} = \Pi + B \tag{6.9}$$

[8]Note that both B (see equation (5.15)) and Π (from its definition (6.7)) are positive for supersymmetric configurations.

for a supertube. Neither Π nor B is invariant under background gauge transformations. However, their worldspace integrals are gauge-invariant.[9] Given the assumption of x-independence, we can identify the IIA string charge q_s and the D0-brane charge per unit length q_0 carried by the supertube probe with the integrals of Π and B over φ, as in (2.16). For constant Π and B we therefore have

$$\Pi = q_s\,, \qquad B = q_0\,. \tag{6.10}$$

The probe supertube tension is then seen to be

$$\tau_{probe} = q_s + q_0\,. \tag{6.11}$$

Setting $\mathcal{E} = U^{-1}$ in (6.7) we deduce that

$$\Pi B = r^2 \tag{6.12}$$

at the supertube radius, and hence that

$$r = \sqrt{q_s q_0}\,. \tag{6.13}$$

The tension and the radius of a supertube probe in the supertube spacetime are therefore exactly as in a Minkowski background (with $N = 1$). Note that this result holds regardless of whether or not CTCs are present.

Thus, supersymmetry places no restriction on the possible radius of a probe supertube in a supergravity supertube background. However, the supertube minimizes the energy for given q_s and q_0 (and hence is stable) only if there are no ghost excitations of the D2-brane, that is, excitations corresponding in the quantum theory to negative norm states. These states *are* present for some choice of q_s and q_0 whenever $g_{\varphi\varphi} < 0$. To establish this, we now allow for time-dependent r and ρ^a, and we expand the Lagrangian density in powers of

$$v^2 = \dot{r}^2 + \delta_{ab}\,\dot{\rho}^a\dot{\rho}^b\,. \tag{6.14}$$

We find[10]

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2}Mv^2 + \cdots\,, \tag{6.15}$$

where $\mathcal{L}_0$ is the Lagrangian density of (6.6), and

$$M = \frac{B^2 + 2U^{-1}fB + U^{-1}Vr^2}{U^{-1/2}V^{1/4}\sqrt{-\det(g+\mathcal{F})}}\,, \tag{6.16}$$

[9] We should consider only time-independent gauge transformations because the Hamiltonian is not expected to be invariant under time-dependent gauge transformations. We should also consider only x-independent gauge transformations to be consistent with our assumption of x-independence.

[10] The calculation here is similar to the one performed in [11] but the interpretation is different.

which reduces to

$$M = \frac{B^2 + 2U^{-1}fB + U^{-1}Vr^2}{U^{-1}V^{1/2}B} \tag{6.17}$$

for a supertube. In both of these expressions for M, the denominator is positive, and so is the numerator as long as $g_{\varphi\varphi} > 0$, but the factor

$$B^2 + 2U^{-1}fB + U^{-1}Vr^2 \tag{6.18}$$

becomes negative for some choices of B and r (and hence q_0 and q_s for a supertube) whenever $g_{\varphi\varphi} < 0$.

Note that the supertube does *not* become tachyonic; its tension continues to be given by (6.11). A tachyon instability leads to runaway behaviour in which the kinetic energy increases at the expense of potential energy. The instability here is instead due to the possibility of negative kinetic energy, characteristic of a ghost.[11] This instability indicates that it is not physically possible to construct a supertube spacetime with naked CTCs starting from Minkowski space. One might imagine assembling such a spacetime by accretion from infinity of 'supertube shells' carrying infinitesimal fractions of charges and angular momentum. When a finite macroscopic fraction of charges and angular momentum have been accumulated, it is fully justified to treat the next shell as a probe in the background generated by the rest. The instability on this probe when the bound (2.29) is violated signals that this procedure must be physically forbidden beyond the bound (2.29).

7. Discussion

We have found the exact IIA supergravity solution corresponding to a source provided by the D2-brane supertube found in [1]. We have also found multi-tube solutions, which shows that parallel supertubes exert no force on each other, and we have confirmed this by showing that when (suitably aligned) stationary D0-brane, IIA-string and D2-supertube probes are introduced into the background, they preserve all supersymmetries of the background. This provides confirmation from supergravity of the matrix model results of [2] for multi-supertubes.

The supertube spacetime reproduces with remarkable accuracy the worldvolume analysis of [1]. It carries the appropriate charges as measured by surface integrals at infinity: string charge Q_s, D0 charge per unit length Q_0 and angular momentum J. Its tension is $\tau = |Q_s| + |Q_0|$, as in [1], and it also preserves the same eight supercharges (1/4 of those of the IIA Minkowski vacuum). It is singular exactly on a

[11]As representations of the Poincaré group, a 'tachyon' is a particle with spacelike energy-momentum vector, and hence negative mass-*squared* (corresponding in the quantum theory to an excitation about a vacuum that is a local maximum of the energy rather than a local minimum). A 'ghost' is a particle with non-spacelike energy-momentum vector but negative energy; at the level of the particle Lagrangian (or, more generally, brane Lagrangian, as discussed here) this corresponds to negative *mass*.

cylindrical surface of radius R (but is regular everywhere else). The only modification relative to the analysis in [1] comes from considering the possibility that the D2-brane winds N times around the tube. Taking this into account, all the formulas for the supertube are precisely reproduced. The radius is related to the angular momentum as $|J| = NR^2$. Finally, the requirement that there are no closed timelike curves in the supertube spacetime imposes the bound $|J| \leq |Q_s Q_0|/N$ on the angular momentum, which is the same as in [1], once multiwinding is allowed for.

If $|J|$ exceeds the bound above then the Killing vector field ℓ, associated to rotations in the plane of the angular momentum, becomes timelike sufficiently close to the tube region, which leads to a global violation of causality. Although this has no effect on the *local* physics of D0-brane or IIA string probes, it causes an instability of D2-supertube probes due to the fact that the supertube tension no longer minimizes the energy for fixed D0 and string charges. This is made possible by the appearance of a negative-norm state (a ghost) on a supertube in the region where ℓ becomes timelike. The global violation of causality that this produces (due to the occurrence of CTCs) is thus manifested by a *local* pathology on supertube probes (which is possible because these probes are themselves non-local). This constitutes evidence that a globally causality-violating supertube spacetime cannot be physically assembled by starting with flat space and continuously bringing in from infinity 'supertube shells' with infinitesimal fractions of charge and angular momentum.

It is straightforward to modify the IIA supergravity supertube solution to one that provides the fields for a supertube in a Kaluza-Klein vacuum spacetime of the form $\mathbb{E}^{(1,n)} \times \mathcal{M}_{9-n}$, $n \geq 4$, with $\mathcal{M}_{9-n}$ a compact Ricci-flat manifold. The metric then takes the form

$$\begin{aligned} ds^2_{10} &= -U^{-1}V^{-1/2}(dt - A)^2 + U^{-1}V^{1/2}dx^2 + \\ &\quad + V^{1/2}[dr^2 + r^2 d\varphi^2 + d\rho^2 + \rho^2 d\Omega^2_{n-4} + ds^2(\mathcal{M}_{9-n})] \,. \end{aligned} \tag{7.1}$$

For even n the solutions involve elliptic functions, and therefore are somewhat awkward to work with. For odd n, instead, they take simple forms. For $n = 7$ (a supertube in eight dimensions),

$$\begin{aligned} U &= 1 + \frac{|Q_s|}{4\Omega_5} \frac{r^2 + \rho^2 + R^2}{\Sigma^3} \,, \\ A &= \frac{J}{2\Omega_5} \frac{r^2}{\Sigma^3}\, d\varphi \,, \end{aligned} \tag{7.2}$$

while for $n = 5$,

$$\begin{aligned} U &= 1 + \frac{|Q_s|}{2\Omega_3 \Sigma} \,, \\ A &= \frac{J}{2\Omega_3} \frac{r^2}{\Sigma(r^2 + \rho^2 + R^2 + \Sigma)}\, d\varphi \,. \end{aligned} \tag{7.3}$$

V is as U with Q_s replaced by Q_0, and Σ is again given by (3.17). In all cases it is possible to verify that the absence of CTCs implies the bound (2.29).

The maximally rotating six-dimensional ($n = 5$) solution is in fact dual to the solution for a helical D-string constructed in [4]. One of the two methods applied in [4] to the construction of this solution (the one based on the chiral null model) is essentially equivalent to the one employed in this paper. The other approach, which starts from the neutral rotating black hole and subjects it to several transformations, does not lead to supertubes in dimensions higher than six. Instead, it can be seen to result in filled-in cylinders, that is, continuous distributions of concentrical tubes inside a cylinder. These solutions do not possess CTCs either provided that J does not exceed the bound.

This six-dimensional supertube is particularly interesting when the compact four-dimensional space is $K3$. Although at weak string coupling the supertube source is distributional, and hence singular, one might expect it to be non-singular at strong coupling. To examine this possibility we should look for solutions of the six-dimensional supergravity/Yang-Mills theory that governs the low-energy limit of the T^4-compactified S-dual heterotic string theory. The distributional D2-brane of the IIA theory now appears as a non-singular 2-brane with a magnetic monopole core. Conceivably, a tubelike configuration of this monopole 2-brane can be supported against collapse by angular momentum in just such a way that its effective worldvolume description is as a D=6 supertube (that is, a d2-brane supertube of iia 'little' string theory). In this case, one might hope to find a 'full' non-singular supergravity/SYM solution that reproduces the fields of the dualized IIA supergravity tube (after the redefinition of fields required by the IIA/heterotic duality) but completed in the interior by a solution with the 'SYM-supertube' source. The case of the heterotic dyonic instanton [12] provides a model for this kind of non-singular completion of a rotating brane solution of supergravity, although in that case the angular momentum is fixed by the charges rather than just being bounded by them.

In view of these connections, it appears that the supergravity supertube spacetime studied here will eventually take its place in a more general theory of supergravity solutions for supersymmetric sources supported by angular momentum.

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